

8-5 Study Guide and Intervention

Using the Distributive Property

Use the Distributive Property to Factor The Distributive Property has been used to multiply a polynomial by a monomial. It can also be used to express a polynomial in factored form. Compare the two columns in the table below.

Multiplying	Factoring
$3(a + b) = 3a + 3b$	$3a + 3b = 3(a + b)$
$x(y - z) = xy - xz$	$xy - xz = x(y - z)$
$6y(2x + 1) = 6y(2x) + 6y(1)$ $= 12xy + 6y$	$12xy + 6y = 6y(2x) + 6y(1)$ $= 6y(2x + 1)$

Example 1: Use the Distributive Property to factor $12mp + 80m^2$.

Find the GCF of $12mp$ and $80m^2$.

$$12mp = 2 \cdot 2 \cdot 3 \cdot m \cdot p$$

$$80m^2 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot m \cdot m$$

$$\text{GCF} = 2 \cdot 2 \cdot m \text{ or } 4m$$

Write each term as the product of the GCF and its remaining factors.

$$\begin{aligned} 12mp + 80m^2 &= 4m(3 \cdot p) + 4m(2 \cdot 2 \cdot 5 \cdot m) \\ &= 4m(3p) + 4m(20m) \\ &= 4m(3p + 20m) \end{aligned}$$

$$\text{Thus } 12mp + 80m^2 = 4m(3p + 20m).$$

Example 2: Factor $6ax + 3ay + 2bx + by$ by grouping.

$$\begin{aligned} 6ax + 3ay + 2bx + by &= (6ax + 3ay) + (2bx + by) \\ &= 3a(2x + y) + b(2x + y) \\ &= (3a + b)(2x + y) \end{aligned}$$

Check using the FOIL method.

$$\begin{aligned} (3a + b)(2x + y) &= 3a(2x) + (3a)(y) + (b)(2x) + (b)(y) \\ &= 6ax + 3ay + 2bx + by \checkmark \end{aligned}$$

Exercises

Factor each polynomial.

1. $24x + 48y$ $24(x + 2y)$

4. $9x^2 - 3x$ $3x(3x - 1)$

7. $14t^3 - 42t^5 - 49t^4$ $7t^3(2 - 6t^2 - 7t)$

10. $4x + 12x^2 + 16x^3$ $4x(1 + 3x + 4x^2)$

13. $x^2 + 2x + x + 2$ $(x + 1)(x + 2)$

16. $12ax + 3xz + 4ay + yz$ $(3x + y)(4a + z)$

2. $30mp^2 + m^2p - 6p$ $p(30mp + m^2 - 6)$

5. $4m + 6p - 8mp$ $2(2m + 3p - 4mp)$

8. $55p^2 - 11p^4 + 44p^5$ $11p^2(5 - p^2 + 4p^3)$

11. $4a^2b + 28ab^2 + 7ab$ $ab(4a + 28b + 7)$

14. $6y^2 - 4y + 3y - 2$ $(2y + 1)(3y - 2)$

17. $12a^2 + 3a - 8a - 2$ $(3a - 2)(4a + 1)$

3. $q^4 - 18q^3 + 22q$ $q(q^3 - 18q^2 + 22)$

6. $45r^3 - 15r^2$ $15r^2(3r - 1)$

9. $14y^3 - 28y^2 + y$ $y(14y^2 - 28y + 1)$

12. $6y + 12x - 8z$ $2(3y + 6x - 4z)$

15. $4m^2 + 4mp + 3mp + 3p^2$ $(4m + 3p)(m + p)$

18. $xa + ya + x + y$ $(a + 1)(x + y)$

$$13.) (x^2 + 2x) + (x + 2)$$

$$x(x+2) + 1(x+2)$$

$$\boxed{(x+1)(x+2)}$$

$$14.) (6y^2 - 4y) + (3y - 2)$$

$$2y(3y-2) + 1(3y-2)$$

$$\boxed{(2y+1)(3y-2)}$$

$$15.) (4m^2 + 4mp) + (3mp + 3p^2)$$

$$4m(m+p) + 3p(m+p)$$

$$\boxed{(4m+3p)(m+p)}$$

$$16.) (12ax + 3xz) + (4ay + yz)$$

$$3x(4a+z) + y(4a+z)$$

$$\boxed{(3x+y)(4a+z)}$$

$$17.) (12a^2 + 3a) + (-8a - 2)$$

$$3a(4a+1) - 2(4a+1)$$

$$\boxed{(3a-2)(4a+1)}$$

$$18.) (xa + ya) + (x + y)$$

$$a(x+y) + 1(x+y)$$

$$\boxed{(a+1)(x+y)}$$

8-5 Study Guide and Intervention *(continued)*

Using the Distributive Property

Solve Equations by Factoring The following property, along with factoring, can be used to solve certain equations.

Zero Product Property	For any real numbers a and b , if $ab = 0$, then either $a = 0$, $b = 0$, or both a and b equal 0.
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Example : Solve $9x^2 + x = 0$. Then check the solutions.

Write the equation so that it is of the form $ab = 0$.

$9x^2 + x = 0$	Original equation
$x(9x + 1) = 0$	Factor the GCF of $9x^2 + x$, which is x .
$x = 0$ or $9x + 1 = 0$	Zero Product Property
$x = 0$ $x = -\frac{1}{9}$	Solve each equation.

The solution set is $\{0, -\frac{1}{9}\}$.

Check Substitute 0 and $-\frac{1}{9}$ for x in the original equation.

$9x^2 + x = 0$	$9x^2 + x = 0$
$9(0)^2 + 0 \stackrel{?}{=} 0$	$9(-\frac{1}{9})^2 + (-\frac{1}{9}) \stackrel{?}{=} 0$
$0 = 0 \checkmark$	$\frac{1}{9} + (-\frac{1}{9}) \stackrel{?}{=} 0$
	$0 = 0 \checkmark$

Exercises

Solve each equation. Check your solutions.

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|--|--|--|
| 1. $x(x + 3) = 0$
$x = 0, -3$ | 2. $3m(m - 4) = 0$
$m = 0, 4$ | 3. $(r - 3)(r + 2) = 0$
$r = -2, 3$ |
| 4. $3x(2x - 1) = 0$
$x = 0, \frac{1}{2}$ | 5. $(4m + 8)(m - 3) = 0$
$m = -2, 3$ | 6. $5t^2 = 25t$
$t = 0, 5$ |
| 7. $(4c + 2)(2c - 7) = 0$
$c = -\frac{1}{2}, \frac{7}{2}$ | 8. $5p - 15p^2 = 0$
$p = 0, \frac{1}{3}$ | 9. $4y^2 = 28y$
$y = 0, 7$ |
| 10. $12x^2 = -6x$
$x = -\frac{1}{2}, 0$ | 11. $(4a + 3)(8a + 7) = 0$
$a = -\frac{7}{8}, -\frac{3}{4}$ | 12. $8y = 12y^2$
$y = 0, \frac{2}{3}$ |
| 13. $x^2 = -2x$
$x = -2, 0$ | 14. $(6y - 4)(y + 3) = 0$
$y = -3, \frac{2}{3}$ | 15. $4m^2 = 4m$
$m = 0, 1$ |
| 16. $12x = 3x^2$
$x = 0, 4$ | 17. $12a^2 = -3a$
$a = -\frac{1}{4}, 0$ | 18. $(12a + 4)(3a - 1) = 0$
$a = \frac{1}{3}$ |

$$1.) x(x+3)=0$$

$$\boxed{x=0} \text{ or } x+3=0$$

$$\boxed{x=-3}$$

$$2.) 3m(m-4)=0$$

$$3m=0 \text{ or } m-4=0$$

$$\boxed{m=0} \text{ or } \boxed{m=4}$$

$$3.) (r-3)(r+2)=0$$

$$r-3=0 \text{ or } r+2=0$$

$$\boxed{r=3} \text{ or } \boxed{r=-2}$$

$$4.) 3x(2x-1)=0$$

$$3x=0 \text{ or } 2x-1=0$$

$$\boxed{x=0} \text{ or } 2x=1$$

$$\boxed{x=\frac{1}{2}}$$

$$5.) (4m+8)(m-3)=0$$

$$4m+8=0 \text{ or } m-3=0$$

$$4m=-8 \text{ or } \boxed{m=3}$$

$$\boxed{m=-2}$$

$$6.) 5t^2=25t$$

$$5t^2-25t=0$$

$$5t(t-5)=0$$

$$5t=0 \text{ or } t-5=0$$

$$\boxed{t=0} \text{ or } \boxed{t=5}$$

$$7.) (4c+2)(2c-7)=0$$

$$4c+2=0 \text{ or } 2c-7=0$$

$$4c=-2 \text{ or } 2c=7$$

$$\boxed{c=-\frac{1}{2}} \text{ or } \boxed{c=\frac{7}{2}}$$

$$8.) 5p-15p^2=0$$

$$5p(1-3p)=0$$

$$5p=0 \text{ or } 1-3p=0$$

$$\boxed{p=0} \text{ or } 1=3p$$

$$\boxed{\frac{1}{3}=p}$$

$$9.) 4y^2=28y$$

$$4y^2-28y=0$$

$$4y(y-7)=0$$

$$4y=0 \text{ or } y-7=0$$

$$\boxed{y=0} \text{ or } \boxed{y=7}$$

$$10.) 12x^2=-6x$$

$$12x^2+6x=0$$

$$6x(2x+1)=0$$

$$6x=0 \text{ or } 2x+1=0$$

$$\boxed{x=0} \text{ or } 2x=-1$$

$$\boxed{x=-\frac{1}{2}}$$

$$11.) (4a+3)(8a+7)=0$$

$$4a+3=0 \text{ or } 8a+7=0$$

$$4a=-3 \text{ or } 8a=-7$$

$$\boxed{a=-\frac{3}{4}} \text{ or } \boxed{a=-\frac{7}{8}}$$

$$12.) 8y=12y^2$$

$$0=12y^2-8y$$

$$0=4y(3y-2)$$

$$4y=0 \text{ or } 3y-2=0$$

$$\boxed{y=0} \text{ or } 3y=2$$

$$\boxed{y=\frac{2}{3}}$$

$$13.) x^2 = -2x$$

$$x^2 + 2x = 0$$

$$x(x+2) = 0$$

$$\boxed{x=0} \text{ or } \boxed{x+2=0}$$

$$\boxed{x=-2}$$

$$14.) (6y-4)(y+3) = 0$$

$$6y-4=0 \text{ or } y+3=0$$

$$6y=4 \text{ or } \boxed{y=-3}$$

$$\boxed{y=\frac{2}{3}}$$

$$15.) 4m^2 = 4m$$

$$4m^2 - 4m = 0$$

$$4m(m-1) = 0$$

$$4m=0 \text{ or } m-1=0$$

$$\boxed{m=0} \text{ or } \boxed{m=1}$$

$$16.) 12x = 3x^2$$

$$0 = 3x^2 - 12x$$

$$0 = 3x(x-4)$$

$$3x=0 \text{ or } x-4=0$$

$$\boxed{x=0} \text{ or } \boxed{x=4}$$

$$17.) 12a^2 = -3a$$

$$12a^2 + 3a = 0$$

$$3a(4a+1) = 0$$

$$3a=0 \text{ or } 4a+1=0$$

$$\boxed{a=0} \text{ or } \boxed{4a=-1}$$

$$\boxed{a=-\frac{1}{4}}$$

$$18.) (12a+4)(3a-1) = 0$$

$$12a-4=0 \text{ or } 3a-1=0$$

$$12a=4 \text{ or } 3a=1$$

$$a=\frac{1}{3} \text{ or } a=\frac{1}{3}$$